EXPERIMENTAL STUDY OF THE VELOCITY OF THE FLOW OF HIGHLY VISCOUS NEWTONIAN LIQUID IN A CURVED RECTANGULAR DUCT

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Thermistor anemometer measurements in a curved rectangular duct and a straight circular cross section tube permitted verification of the theoretical values of tangential velocities computed on the basis of the solution of the Navier-Stokes equation for the drag isothermal and creeping flow of a Newtonian liquid. From comparison of the theoretical and experimental values there follows that the achieved agreement is acceptable.

There are numerous experimental techniques and methods available to measure local velocities of liquid flow in various flow situations. These have been well described in the literature: The Pitot Tube[1], hot wire anemometer[2], flow visualisation techniques with photocatalytic dye reacting to ultraviolet rays[3], stereoscopic motion picture technique[4], laser technique[5], *etc.*

A thermistor anemometer[6] has been developed in the Institute of Chemical Equipment to measure the velocity of flow of highly viscous liquid in a mixed vessel.

Although the aim of our work was merely calibration of the above instrument whose probe permits measurements of only absolute values of the velocity of the flow and not its direction, the obtained results did bring valuable knowledge of the flow of liquids in a curved duct of rectangular cross section. Theoretical solution of the given problem, encompassing the region of creeping flow of Newtonian liquids, has been proposed by Rieger and Šesták[7] both for the limiting case of purely pressure flow and purely drag flow, and for the general case of combined drag and pressure flow.

The results of the solution presented in ref.[7] lead to the design of a certain calibration system of the thermistor anemometer. The second system was then based on the conditions of the creeping flow in tube.

It should be noted that systems used in this work with a freeboard of liquid proved unsuitable for the calibration owing to the permanent formation of "skin" on the measured highly viscous liquid containing water. This phenomenon caused considerable decrease of the velocity of the flow in the proximity of the probe.

Comparison of results of both calibration systems offers the possibility to estimate the accuracy of the theoretical solution of the flow of highly viscous liquid under the conditions corresponding to the experimental arrangement.

THEORETICAL

Consider a pressureless drag flow of a highly viscous Newtonian liquid in a curved or annular duct of rectangular cross section shown schematically in Fig. 1. The Navier–Stokes equations for the given arrangement of the flow in cylindrical co-ordinates reduce to the following form[8]

$$\partial^2 v_\Theta/\partial r^2 + (1/r)(\partial v_\Theta/\partial r) - v_\Theta/r^2 + \partial^2 v_\Theta/\partial z^2 = 0\,, \tag{1}$$

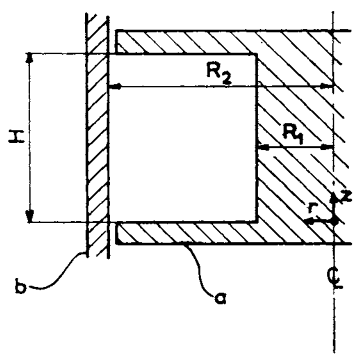

FIG. 1

Cut view of a curves annular duct of rectangular cross section *a* immobile part, *b* rotating wall of the duct

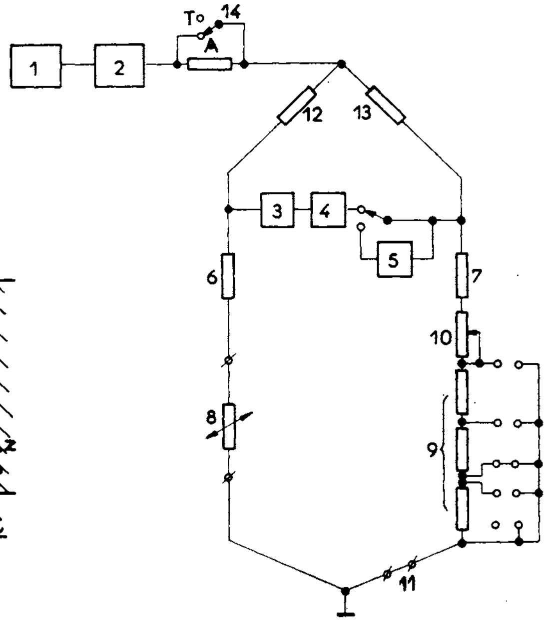

FIG. 2

Block diagram of thermistor anemometer. **1** Power supply, **2** stabilizer, **3** bridge, **4** range switch, **5** source of compensation voltage, **6** shunt for measurement of the current through the probe, **7** resistor, **8** probe, **9** coarse bridge balancing, **10** fine bridge balancing, **11** connectors for external circuit, **12, 13** resistors of the upper branch of the bridge, **14** mode switch: A anemometer, T thermometer with the initial conditions (Fig. 1)

$$\begin{aligned} z &= 0 & v_\theta &= 0 \\ z &= H & v_\theta &= 0 \\ r &= R_1 & v_\theta &= 0 \\ r &= R_2 & v_\theta &= 2\pi R_2 n\,. \end{aligned} \tag{2}$$

In the given system thus there exists only the tangential velocity component $v_\theta(r, z)$. The analytical solution of the set of equations (*1*) and (*2*), including the equations of continuity for a incompressible fluid and isothermal conditions of the flow, has been found by Rieger and Šesták[1] in the form

$$\phi(\varrho', \xi) = 2 \sum_{j=1,3,5,\ldots}^{\infty} [A_1(j)\, I_1(\varrho') + B_1(j)\, K_1(\varrho')] \sin(j\pi\xi)\,, \tag{3}$$

where

$$\varrho' = j\pi(R_2/H)\,[R_1/R_2 + (1 - R_1/R_2)\,\eta] \tag{4a}$$

$$\eta = (r - R_1)/(R_2 - R_1) \tag{4b}$$

$$\xi = z/H \tag{4c}$$

$$\phi = v_\theta/2\pi R_2 n \tag{4d}$$

and further

$$A_1(j) = 2\mathrm{K}_1[(R_1/H)\, j\pi]/j\pi F(n) \tag{5a}$$

$$B_1(j) = 2\mathrm{I}_1[(R_1/H)\, j\pi]/j\pi F(n) \tag{5b}$$

$$F(n) = \mathrm{I}_1(j\pi R_1/H)\,\mathrm{K}_1(j\pi R_1/R_2) - \mathrm{I}_1(j\pi R_1/R_2)\,\mathrm{K}_1(j\pi R_1/H)\,. \tag{6}$$

I_1, K_1 in these expressions are corresponding Bessel functions. The relation (*3*) determines the dependence of the tangential component of velocity on the radial and axial coordinate.

The second system considered, the creeping flow of liquid in a straight tube of circular cross section, under the same conditions as in the above case, is characterized by the parabolic profile of axial velocity

$$v_z(r) = (2\dot{V}/\pi R^2)\,[1 - (r/R)^2]\,, \tag{7}$$

while on the axis of the tube of internal diameter $d_t = 2\mathrm{R}$ we have

$$v_z(r = 0) = 8\dot{V}/\pi d_t^2\,. \tag{8}$$

EXPERIMENTAL

Thermistor anemometer. A glass covered thermistor 13 NR 09/A manufactured by Pramet Šumperk was connected into an anemometric resistance bridge and in parallel also into a temperature bridge. The circuitry of the anomometer is shown schematically in Fig. 2. The anemometric bridge was supplied by a constant current (about 4 mA) while the temperature bridge was supplied by a current approximately 1 000 times less. The bead of the thermistor, approximately 1 mm in diameter, was affixed to a stainless steel tube 2·5 mm in diameter 120 mm long with the aid of an epoxy resin cone. By switching the function of the thermistor one could determine, with a small time lag of a few seconds, the temperature of ambient liquid as well as the steady state conditions of cooling the heated thermistor under the corresponding conditions of the flow past the probe (the change of voltage on the diagonal of the bridge — U_D).

Annular duct of rectangular cross section. The experimental arrangement of the annular duct of rectangular cross section is shown schematically in Fig. 3. The set-up consisted of a turn-table t, driven by a rubber disc r using a transmission gear. The disc was mounted on a shaft s of a laboratory mixer. On the turn-table there was a shallow cylindrical vessel v with a flat bottom. Inside the vessel there was an immobile barrel b shaped as a "reel", equipped on the internal part by circular openings h_1, h_2 for filling the duct of the calibration equipment by a highly viscous liquid. The filling of the duct by the liquid was carried out through circular openings h_3. The upper wall of the duct was equipped with a radial slot g 3 mm wide for inserting and shifting the probe along the duct. The anemometric probe a, terminated with the thermistor bead, could reach in this manner an arbitrary position on the radial cross section of the duct. The rectangular cross section measured 101 × 66·5 mm. During the rotation of the turn table and the vessel the immobile barrel slided over the bottom of the vessel on three half-spherical journals p. Rotation of the table was being adjusted by continuously variable speed of revolution of the shaft and checked by a stroboscop.

The measurement of the velocity in the duct was carried out exclusively in the upper half of the rectangular cross section in order to avoid the effect of the lower filling openings h_3.
The measurements consisted of determining the values of the voltage, U_D, on the diagonal of the anemometric bridge. This value corresponded to the increased heat loss from the heated probe due to the flow of ambient liquid in comparison with the "quiescent" situation ($U_D = 0$) when the vessel was at still (cooling by natural convection).

Prior to the measurement the liquid in the duct was freed of bubbles through the slot g for shifting the probe a. The temperature of the liquid was checked during measurement by the

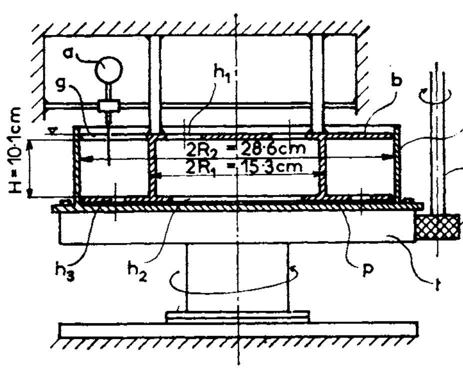

FIG. 3

Sketch of experimental set-up to measure velocities in an annular duct of reactangular cross section. a Thermistor probe, b immobile internal part of the duct, g slot for shifting the probe in the duct, h_1, h_2, h_3 openings for filling the duct with highly viscous liquid, p ball journal, r rubber disc driving the turn-table s driving shaft, t turn-table, v rotating vessel thermistor. The measurements during which the temperature change inside the duct exceeded 0·5 K were scratched.

System with a tube of circular cross section. The measuring loop with the straight tube of circular cross section is shown schematically in Fig. 4. It is apparent that the highly viscous liquid discharged from the bottom part of the constant head storage tank v into a tube of circular cross section t of the total length 1·2 m. PVC tubes were used for measurements with internal diameters 2·02 cm and 3·34 cm. The anemometric probe was placed perpendicularly to the direction of the flow so that the thermistor bead was always in the axis of the tube. The liquid was supplied into the storage tank v by a pump p and the constant pressure at the inlet end of the tube was maintained by an overflow weir o and measured by a U-manometer u. Values of the flow rate of liquid through the tube $\dot{V}$ were determined volumetrically (past the valve b_3). The anemometric measurements again consisted of determining (for the given flow rate of liquid) of measuring the voltages U_D. Zeroing of the anemometric bridge ($U_D = 0$) was carried out again with imersed probe in the still liquid (valve b_3 shut). The value of the flow rate was accomplished by a change of air pressure in the storage tank. Simultaneously with the measurement of the velocity of the flow also temperatures of the liquid were checked by swithing the function and supply of the thermistor.

Liquids used. For measurements in the annular duct of rectangular cross section in each measuring position (coordinate r, z) the velocity $v_\theta(r, z)$ was computed from Eqs (*3*) and (*6*) on a computer for constant frequency of revolution of the vessel, $n = 0{\cdot}5\ \mathrm{s}^{-1}$. The results, in the form of the dependence $U_D = U_D(v_\theta)$ are shown for all four liquids in Figs 5 and 6.

TABLE I

Physical properties of liquids used

Designation	t, °C	μ Pa s	ϱ kg m^{-3}	c_p J kg^{-1} K^{-1}	λ W m^{-1} K^{-1}
I	20·5*	9·50	1 418	2 160	0·337
II	20·5*	2·75	1 401	2 182	0·344
III	20·5*	0·89	1 352	2 320	0·360
IV	21·1	0·52	1 316	2 470	0·378

* Values shown in Figs 5 and 6 correspond to a changed temperature of liquid.

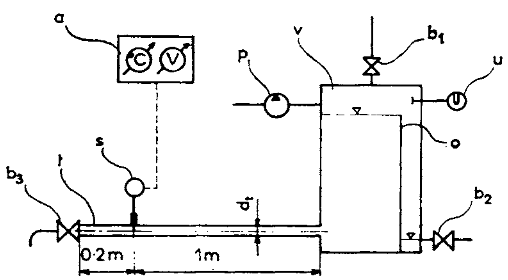

FIG. 4

Sketch of experimental set-up to measure velocities in a tube of circular cross section. a Anemometer and thermometer, b_1 pressure air valve, b_2 discharge liquid valve, b_3 closing valve, o overflow, p pump, s thermistor probe, t measuring tube, u U-manometer, v liquid storage tank

The measurements on both test set-ups were realized with water solutions of starch exhibiting Newtonian behaviour. The physical properties of the liquids are summarized in Table I. The values from this table were determined in the physico-chemical laboratory of the Institute of chemical equipment (*e.g.* dynamic viscosity on the rotational viscometer RHEOTEST RV, density by pycnometry).

RESULTS

The overall course of the curves $U_D(v_\theta)$ is similar for all liquids used. However, for individual liquids the curves shift toward higher values of U_D for higher dynamic viscosities μ. For instance, for the velocity $v_\theta = 15\ \mathrm{cm\ s^{-1}}$ the values of U_D for individual liquids are as follows: IV – $U_D \doteq 3{\cdot}35$ V, III – $U_D \doteq 3{\cdot}6$ V, II – $U_D \doteq 4{\cdot}0$ V, I – $U_D \doteq 4{\cdot}2$ V (values read off the curves). Experimental values of U_D for calculated velocities in different positions of the probe in the duct can be fitted for each liquid in the whole range of measurements by a single curve with acceptable accuracy. An exception to this is only the measurement with the least immersion of the probe, *i.e.* $z' = 0{\cdot}5$ cm for all four liquids and for $z' = 1{\cdot}0$ cm

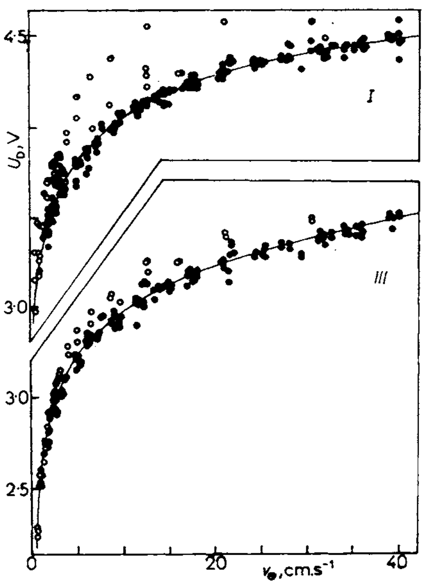

FIG. 5

Plot of the function $U_D = U_D(v_\theta)$ for drag flow in an annular duct. I $\mu = 8{\cdot}40$ Pa s, III $\mu = 0{\cdot}85$ Pa s, immersion depth of the probe z': ○ 0·5 cm, ⦶ 1·0 cm, ⊖ 1·5 cm, ⊗ 2·0 cm, ⊕ 2·5 cm, ◒ 3·0 cm, ⊙ 4·0 cm, ● 5·0 cm

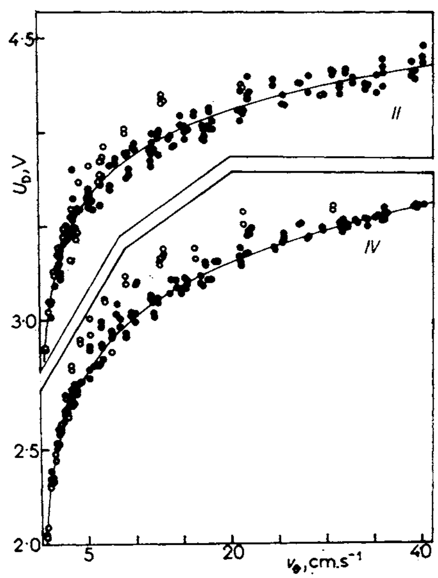

FIG. 6

Plot of the function $U_D = U_D(v_\theta)$ for the drag flow in an annular duct. II $\mu = 2{\cdot}22$ Pa s, IV $\mu = 0{\cdot}52$ Pa s, caption for the depth of immersion same as in Fig. 5

for two liquids with the lowest viscosity. In these cases the measured voltages U_D were higher. The curves $U_D(v_\theta)$ from Figs 5 and 6 were further used as "calibration" curves to evaluate results of measurements on the set-up with the tube of circular cross section. Owing to the low values of the Reynolds criterion for the flow ($Re_{max} = 2{\cdot}3$)

$$Re = vd_t\varrho/\mu \tag{9}$$

axial velocities of the flow past the probe from Eq. (*8*) were used for evaluation. For each value of U_D measured by the probe in the tube a corresponding velocity of the flow, v_θ, was read off the Fig. 5 and 6 for the flow in the annular duct. The set of experimental values of the velocity of the flow in the tube, v_z, and corresponding velocities in the annular duct, v_θ (the same value of U_D in both cases), was subjected in the form of the correlation

$$v_\theta = b_0 + b_1 v_z \tag{10}$$

to a regression analysis. The value of the regression coefficient b_0 proved to be statistically insignificant at the significance level of 0·05. After omitting the coefficient b_0 the following correlation was obtained

$$v_\theta = 0{\cdot}914\ v_z \tag{11}$$

with the following regression parameters:
The significance test of b_1: $S_{b1} = 32{\cdot}04$; the variance of R: $\sigma_R^2 = 4{\cdot}27$; the standard deviation: $S_R = 2{\cdot}07$; the correlation coefficient: $r = 0{\cdot}998$.

The statistical analysis of the experimental and "calculated" velocities shows that the calculated values v_θ are approximately 10% lower than the velocities measured in the annular duct.

DISCUSSION

Measurement of the dependences $U_D(v_\theta)$ for four viscous liquids (Figs 5 and 6) display an acceptable scatter around the computed curve. This fact evidences that the velocities predicted according to the theoretical solution of Rieger and Šesták[1] in various positions of the duct qualitatively correspond to the measured velocities. Quantitatively, however, one cannot overlook the systematic deviation of the measured and computed velocities of the flow. The scatter of the measured data may be attributed to the sensitivity of the anometer and partly also to the imperfect isothermality of the flow of liquid in the annular duct and tube.

The deviating (higher) values of the probe at the point $z' = 0{\cdot}5$ cm and partly also at the point $z' = 1{\cdot}0$ cm may be explained also by the inaccuracy of the estimate of the depth of immersion of the probe ($\pm 0{\cdot}05$ cm) and also by partial non-parallel allignment of the bottom of the vessel and the upper part of the barrel. Otherwise the values of U_D measured at various points of the duct (changing r, z') do not display systematic deviation from the fitted curves $U_D(v_\Theta)$.

The course of the "calibration" curves proper seems to suggest that the values of U_D and hence the values of the electrical resistance of the termistor increase with increasing dynamic viscosity of the flowing liquid. In view of the decrease of the resistance of the thermistor with an increase of its temperature this phenomenon would require a detailed analysis of the conditions of heat removal from the bead of the heated thermistor. Such analysis, however, exceeds in scope this contribution. It may be assumed though that the dependence of the heat transfer coefficient α from the thermistor into a flowing highly viscous liquid is governed by the relationship[9]

$$\alpha = \phi_1 + \phi_2 \sqrt{v}\,, \tag{12}$$

where ϕ_1, ϕ_2 depend on physical properties of the flowing liquid (Table I), temperature of the thermistor and the liquid. The relationship between the coefficient α and the voltage U_D is considerably complex and, moreover, involves to some extent also the conditions of free convection at zeroing the anemometric bridge prior to the velocity measurement. The overall course of the $U_D(v_\Theta)$ curves, however, seems to correspond to Eq. (*12*).

The results of measurements of the velocity of the flow in the straight tube at the point of maximum velocity exhibit considerable scatter of the measured data, particularly in region of higher velocities ($v_z > 4$ cm s^{-1}). This may be accounted for, appart from the error of the anemometer proper, by

- fluctuations of the velocity of the flow in the tube due to the pressure fluctuations in the liquid storage tank
- deformation of the velocity profile due to the existence of radial and axial temperature profile in the tube
- insufficient slope of the curves $U_D(v_\Theta)$ for velocities $v > 4$ cm s^{-1}.

Comparison of the results of measurement of the velocity of the creeping pressureless flow of a Newtonian liquid in an annular duct of rectangular cross section and a straight tube of circular cross section have shown that the solution of the velocity field from the Navier–Stokes equation published for the former case by Rieger and Šesták yields results whose deviations from real values amount to less than 10%.

LIST OF SYMBOLS

A_1, B_1	functions given by Eqs (*5a*), (*5b*)
d_t	internal diameter of tube
H	height of rectangular cross section duct
I_1	modified Bessel function, first order, first kind
K_1	modified Bessel function, first order, second kind
n	frequency of revolution of vessel
R_1	internal diameter of annular duct
R_2	external diameter of annular duct
R	internal diameter of tube
r	coordinate of radius
U_D	voltage in diagonal of anemometric bridge
$\dot{V}$	volume flow rate of liquid
$\bar{v}$	mean velocity
v_z	axial velocity component, here maximum velocity
v_θ	tangential velocity component, here tangential velocity
z	axial coordinate
z'	depth of immersion of the probe in the duct
α	heat transfer coefficient
μ	dynamic viscosity of liquid
ϱ	density of liquid
ϱ'	transformed radial coordinate, see Eq. (*4a*)
η	dimensionless radial coordinate, see Eq. (*4b*)
ϕ	dimensionless velocity of flow, see Eq. (*4d*)
$\phi_{1,2}$	function in Eq. (*12*)
ξ	dimensionless axial coordinate, see Eq. (*4c*)
Re	Reynolds number

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